3/20/17

"The question isn't 'Who is going to let me?'; It's 'Who is going to stop me?'." -Ayn Rand

HW: "Indefinite Integral" #1-6, 15-18
Test 3 on Thursday 3/30

AIM: What is Integration?

# ANTIDERIVATIVES AND INDEFINITE INTEGRATION

VOCABULARY AND DEFINITIONS:

1. A function *F* (*x*) is an *antiderivative* of a function if $F'(x) = f(x)$ for all *x* in domain of *f* . The process of finding an antiderivative is *antidifferentiation.*

2. The family of all antiderivatives of a function *f* (*x*) is the indefinite integral of *f* with respect to *x* and is denoted by $\int f(x)\,dx$

3. If *F* is any function such that $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$ is called the *general solution* and C is called the *constant of integration* (an arbitrary constant).

General Solution is denoted by:

Variable of Integration

Constant of Integration

$$y = \int f(x)\,dx = F(x) + C$$

Integrand

$\int f(x)\,dx$ read as the *antiderivative of f with respect to x.*

So, the differential *dx* serves to identify *x* as the variable of integration. The term indefinite integral is a synonym for antiderivative.

**EX #3:** Applying Basic Rules

A.) $\int 3x\,dx = \boxed{\frac{3x^2}{2} + C}$

B.) $\int 8\,dx = \boxed{8x + c}$

C.) $\int (x^2 - 3x^1 + 4x^0)\,dx = \boxed{\frac{x^3}{3} - \frac{3x^2}{2} + 4x + c}$

**EX #4:** Rewriting Before Integrating

| Original Integral | Rewrite | Integrate | Simplify |
|---|---|---|---|
| $\int \frac{1}{x^3} dx$ | $\int x^{-3} dx$ | $\frac{x^{-2}}{-2} + c$ | $\frac{1}{-2x^2} + c$ |
| $\int \sqrt{x}\ dx$ | $\int x^{1/2} dx$ | $\frac{x^{3/2}}{3/2} + c$ | $\frac{2\sqrt{x^3}}{3} + c$ |
| $\int 2\sin x\ dx$ | $2\int \sin x\ dx$ | $2(-\cos x) + c$ | $-2\cos x + c$ |

$$2x^3 \longrightarrow 2 \cdot x^3$$

$$2 \cdot 3x^2$$

$$6x^2$$

When integrating, think before you work! Practice will help you "discover" many tricks that make the integrations rules "fit your problems."

| TRICK #1 REWRITE | TRICK #2 MULTIPLE or DISTRIBUTE | TRICK #3 SEPARATE FRACTIONS |
|---|---|---|

EX #5: Integrate the following:

A.) $\int (3x-1)^2\,dx = \int (9x^2-6x+1)\,dx = \frac{9x^3}{3} - \frac{6x^2}{2} + x + c$

$(3x-1)(3x-1)$

$9x^2-6x+1$

$= \boxed{3x^3-3x^2+x+c}$

B.) $\int \frac{x^2+4x+5}{x^5}\,dx = \int (x^{-3}+4x^{-4}+5x^{-5})\,dx$

$\frac{x^2}{x^5} + \frac{4x}{x^5} + \frac{5}{x^5}$

$x^{-3}+4x^{-4}+5x^{-5}$

$= \frac{x^{-2}}{-2} + \frac{4x^{-3}}{-3} + \frac{5x^{-4}}{-4} + c$

$= \boxed{\frac{1}{-2x^2} + \frac{4}{-3x^3} + \frac{5}{-4x^4} + c}$

C.) $\int \frac{3\cos\theta}{2}\,d\theta$

D.) $\int \frac{(2x-3)(3x+4)}{\sqrt{x}}\,dx$

E.) $\int \frac{\sin x}{\cos^2 x} dx$

F.) $\int 5\sqrt[3]{x} - 2e^x \, dx$

EX #6: Solve the differential equations subject to given conditions.

A.) $f'(x) = 2x - 1 \qquad f(2) = 5$

C.) $f'(x) = x^2 + x - 7 \quad f(3) = 1$

D.) $f'(x) = 3x^2 - 4x + 2 \qquad f(-1) = -4$